

# First principles impurity diffusion coefficients

M. Mantina<sup>a,\*</sup>, Y. Wang<sup>a</sup>, L.Q. Chen<sup>a</sup>, Z.K. Liu<sup>a</sup>, C. Wolverton<sup>b</sup>

<sup>a</sup> Department of Materials Science and Engineering, The Pennsylvania State University, University Park, PA 16802, USA

<sup>b</sup> Department of Materials Science and Engineering, Northwestern University, Evanston, IL 60208, USA

Received 10 April 2009; accepted 4 May 2009

Available online 22 June 2009

## Abstract

We report the prediction of impurity diffusion coefficients entirely from first principles, using density-functional theory (DFT) calculations. From DFT we obtain all microscopic parameters in the pre-factor and activation energy of impurity diffusion coefficients: (i) the correlation factor through a five frequency model, (ii) the impurity jump frequency within the framework of transition state theory and (iii) the free energies of vacancy formation and vacancy–solute binding. Specifically, we calculate the impurity diffusion coefficients of Mg, Si and Cu in dilute face-centered cubic Al alloys. The results show excellent agreement with experimental data. We discuss the factors contributing to the trends in diffusivities of these impurities.

Published by Elsevier Ltd. on behalf of Acta Materialia Inc.

**Keywords:** Tracer diffusion of Mg, Si, Cu; Activation energy and diffusion pre-factor; Dilute aluminum alloys; Density-functional theory

## 1. Introduction

During the past two decades there have been many efforts to determine diffusion coefficients using fundamental electronic/atomistic approaches [1–8]. The calculations have been either from empirical or semi-empirical approaches [1,8] or involved approximations for the values of certain quantities [2–7]. Recently we presented [9] a parameter-free first principles procedure to predict vacancy-mediated self-diffusion coefficients in cubic systems within the framework of transition state theory (TST). Here we extend this first principles approach to the case of impurity diffusion in face-centered cubic (fcc) systems using the five frequency model developed by LeClaire and Lidiard [10].

The five frequency model has been previously employed to determine impurity diffusion coefficients in alloys using atomistic/electronic structure calculations. For example, Adams et al. [1] predicted impurity diffusion coefficients in fcc metals using lattice static calculations with embedded atom potentials. Janotti et al. [5] and Kremar et al. [6] pre-

dicted diffusion coefficients of transition elements in fcc Ni via first principles calculations. In these studies the diffusion pre-factors were not explicitly calculated, but approximated to be the same for all solute elements. Recently Simonovic and Sluiter [11] conducted an extensive, systematic study of first principles activation energies in Al across a wide range of impurities. There have also been a number of attempts [6,12–16] to elucidate the trends in solute diffusion coefficients based on factors such as size, valence or solubility of the impurity in the host lattice. In most cases the analysis was mainly based on the relative values of activation energies for diffusion of impurities/solutes.

In the present work we use a first principles approach to calculate both the diffusion pre-factor ( $D_0$ ) and the activation energy ( $Q$ ) of impurity diffusion, including the important contributions of correlation for impurity diffusion. Our first principles calculations include both atomically relaxed static total energies, as well as direct force constant phonon frequencies and vibrational free energies. We consider the well-studied cases of impurity diffusion of Mg, Si and Cu in fcc Al. We examine the trends in solute diffusivities based on both  $D_0$  and  $Q$ . This paper is organized as follows. In Section 2 the basic atomic theory of impurity diffusion is presented and in Section 3 the methodology

\* Corresponding author.

E-mail address: [manjeera.m@gmail.com](mailto:manjeera.m@gmail.com) (M. Mantina).

to compute the microscopic quantities in the five frequency model is detailed. The results and discussion are presented in Sections 4 and 5, respectively.

**2. Equations for impurity diffusion coefficients**

Impurities in fcc metals diffuse predominantly via a vacancy mechanism [17]. To describe impurity diffusion one must of course consider the jump frequency of the impurity atom into a neighboring vacant site. However, in addition, the situation is complicated by the fact that in the presence of an impurity the jump frequencies of host atoms surrounding the impurity are different compared with that in the pure element system, and one must consider each of these distinct atomic jumps to describe impurity diffusion with its correlation factor. Le Claire and Lidiard [10] described diffusion in dilute fcc alloys with dilute vacancy concentration in terms of five jump frequencies as:

$$\frac{D_2}{D_0} = \frac{f_2}{f_0} \frac{w_4}{w_0} \frac{w_1}{w_3} \frac{w_2}{w_1} \quad (1)$$

In this equation  $D_2$  is the diffusion coefficient of the impurity atom in the host lattice,  $D_0$  is the self-diffusion coefficient of the pure host element,  $f_2$  is the correlation factor for impurity diffusion,  $f_0$  is the self-diffusion correlation factor and  $w_j$  ( $j = 0 - 4$ ) are the five jump frequencies as illustrated in Fig. 1.  $w_0$  is the host atom jump in the absence of an impurity,  $w_1$  is the jump for a host atom (nearest neighbor to an impurity) jump which does not “dissociate” the impurity from the vacancy,  $w_2$  is the impurity atom jump,  $w_3$  is the host atom jump which “dissociates” the impurity and vacancy and  $w_4$  is the reverse of jump  $w_3$ , i.e. the host atom jump that “associates” the impurity and vacancy.

The impurity diffusion correlation factor,  $f_2$ , is related to the probability of the impurity atom making a reverse jump

back to its previous position. It includes the probability (function  $F$ ) of the vacancy returning to its position after disassociation by a  $w_3$  jump, and is given by Le Claire [18] in terms of the jump frequencies via the following expression:

$$f_2 = \frac{1 + 3.5(w_3/w_1)F(w_4/w_0)}{1 + (w_2/w_1) + 3.5(w_3/w_1)F(w_4/w_0)} \quad (2)$$

By explicitly considering the probabilities of the vacancy returning from second, third and fourth nearest neighboring positions to its original position,  $F$  was defined by Manning [19] as:

$$F(x) = 1 - \frac{10x^4 + 180.5x^3 + 927x^2 + 1341}{7(2x^4 + 40.2x^3 + 254x^2 + 597x + 435)} \quad (3)$$

where  $x$  is  $w_4/w_0$ .

Based on transition state theory (TST), the atom jump frequency is written as [20];

$$w = v^* \exp(-\Delta H_m/k_B T), \quad (4)$$

where  $\Delta H_m$  is the change in enthalpy of the system between the state in which the diffusing atom is in its initial equilibrium lattice position (hereafter referred to as the initial state) and the state in which it is at the saddle point along the diffusion path (hereafter referred to as the transition state), and  $v^*$  is the effective frequency described by Vineyard [21] as the quotient of the product of vibrational frequencies of the initial state  $v_i$  to that of the non-imaginary frequencies of the transition state  $v'_i$ . For a system with  $N$  vibrational degrees of freedom  $v^*$  is given by:

$$v^* = \frac{\prod_{i=1}^N v_i}{\prod_{i=1}^{N-1} v'_i} \quad (5)$$

As illustrated in Fig. 1, three types of  $w_3$  and  $w_4$  jumps are possible in an fcc lattice, depending on whether the second, third or fourth nearest neighboring site to the impurity is involved in the jump process. The five frequency model assumes that the three types of each jump are equiv-

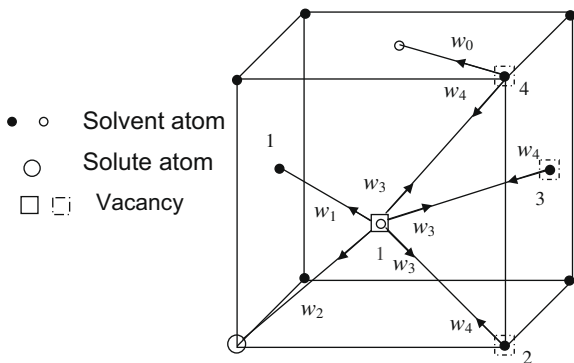


Fig. 1. Five frequency model illustration for the case of an fcc system with a dilute impurity concentration. The arrows indicate the direction of the vacancy jump and the numbers  $n$  stand for the  $n$ th nearest neighboring site to the impurity. For the cases of  $w_1$ ,  $w_2$  and  $w_3$  jumps the vacancy position adjacent to the solute is indicated by a solid box and filled circles indicate solvent atoms that make these jumps. For jumps  $w_0$  and  $w_4$  the vacancy position away from the solute is indicated by a dotted box and the solvent positions for these jumps are indicated by open circles.

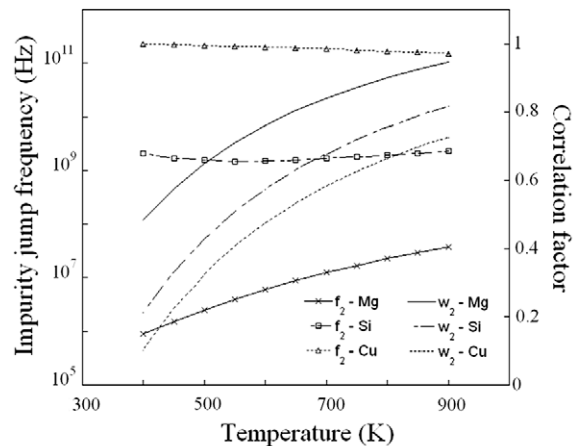


Fig. 2. Impurity jump frequency and correlation factor as a function of temperature of Mg, Si and Cu in Al from LDA.

alent and thus have the same jump frequencies. In addition,  $w_3$  is simply the reverse of  $w_4$  and hence these two jumps have the same transition state, resulting in the expression [22]:

$$\frac{w_4}{w_3} = \exp\left(\frac{\Delta G_b}{k_B T}\right), \quad (6)$$

where  $\Delta G_b$  is the solute–vacancy binding free energy representing the free energy difference between the state where solute and vacancy are nearest neighbors (initial state of  $w_3$  and final state of  $w_4$ ) and the state where they are separate (initial state of  $w_4$  and final state of  $w_3$ ).

Eq. (1) is a valid description of the impurity diffusion coefficient in an fcc system with a dilute impurity concentration. However, it is often convenient to represent this equation in an Arrhenius form to obtain the pre-factor and activation energy of impurity diffusion. Combining the expression describing the self-diffusion coefficient [9],

$$D_0 = f_0 w_0 C_0 a^2 \quad \text{with } C_0 = \exp\left(-\frac{\Delta G_f^\circ}{k_B T}\right),$$

with Eq. (6) we can write Eq. (1) as:

$$D_2 = f_2 w_2 a^2 \exp\left(-\frac{\Delta G_f^\circ - \Delta G_b}{k_B T}\right), \quad (7)$$

where  $a$  is the lattice parameter and  $\Delta G_f^\circ$  is the free energy of vacancy formation in the pure host system. Representing  $\Delta G_f^\circ - \Delta G_b$  as  $\Delta G_f$ , the free energy for vacancy formation minus the solute–vacancy binding [18], along with the associated enthalpy and entropy,  $\Delta H_f$  and  $\Delta S_f$ , and substituting  $w_2 = v^* \exp(-\Delta H_m/k_B T)$ , the impurity diffusion Eq. (7) can be expressed in an Arrhenius form as [22]:

$$D_2 = \frac{f_2 a^2 v^* \exp\left(\frac{\Delta S_f}{k_B}\right)}{\exp\left(\frac{k_B d \ln f_2/d(1/T)}{k_B T}\right)} \times \exp\left(-\frac{(\Delta H_f + \Delta H_m - k_B d \ln f_2/d(1/T))}{k_B T}\right), \quad (8)$$

where the activation energy is given by

$$Q = \Delta H_f + \Delta H_m - k_B d \ln f_2/d(1/T)$$

and the diffusion pre-factor is

$$D_o = \frac{f_2 a^2 v^* \exp\left(\frac{\Delta S_f}{k_B}\right)}{\exp\left(\frac{k_B d \ln f_2/d(1/T)}{k_B T}\right)}.$$

We note that the temperature dependence of  $f_2$  from Eq. (4) contributes to the activation energy of impurity diffusion.

### 3. Methodology

For the first principles calculations we use the Vienna ab initio simulation package [VASP] [23] with projector augmented wave (PAW) potentials [24,25]. The potentials used

for each of the elements Al, Mg, Si and Cu do not treat any semi-core states as valence. We have compared results using both the local density approximation (LDA) [26] and the generalized gradient approximation (GGA) [27] for the exchange correlation. Convergence tests indicate that a Monkhorst–Pack  $k$  point mesh of  $11 \times 11 \times 11$  and an energy cut-off of 300 eV are suitable to yield converged impurity migration barriers within 0.01 eV. Similar convergence of the energetics is obtained in tests using supercells with 32 and 64 lattice sites. Hence, we employ supercells with 32 lattice sites ( $2 \times 2 \times 2$  conventional fcc cells), replacing one host atom with an impurity atom.

The thermodynamic properties of the initial and transition states of a jump need to be determined to calculate its jump frequency ( $w_j$ ) using Eq. (4). Each initial state is completely relaxed with respect to internal coordinates, volume and shape. We quantitatively determine the transition state with the saddle point along the minimum energy diffusion path by nudged elastic band (NEB) [28] calculations. We first tested an eight image versus single image NEB calculation for the  $w_0$  jump, and finding good agreement for these relatively high symmetry jumps we performed simply single image calculations for the other jumps. Conducting NEB calculations is specifically important for the case of jumps  $w_1$ ,  $w_3$  and  $w_4$  with asymmetry between the initial and final states. (For the jumps  $w_0$  and  $w_2$  the initial and final states are equivalent and hence there is high symmetry in the jump geometry. Hence, NEB results often give the obvious guess, i.e. the center point between the initial and final positions of the diffusing atom.) Three of the jumps,  $w_1$ ,  $w_2$  and  $w_3$ , have the same initial state, with the vacancy adjacent to the impurity atom, and two jumps,  $w_3$  and  $w_4$ , have the same transition state since they represent the same atomic jump, just in opposite directions. For the  $w_1$  jump both the initial and transition states of the jump in the pure host system need to be considered.

Phonon frequencies are calculated using the direct force constant supercell approach [29] as implemented in the Alloy Theoretic Automated Toolkit (ATAT) [30] package. The same energy cut-off and  $k$  point mesh size as for total energy calculations are used for the vibrational calculations. We obtain  $v^*$  from Eq. (5) using phonon frequencies from the  $\Gamma$  point wave vector. It has been observed in our previous work [9] that anharmonic effects from volume expansion are negligible for aluminum. Hence, in the present work we adopt the harmonic approximation (HA). The migration barrier is then obtained as the difference between the energies of the relaxed configurations at 0 K ( $E_c$ ) of the initial and transition states of a jump. There exists an error in the energetics of vacancy-containing systems obtained from density-functional theory due to an overestimation of the energy of the vacancy, which may be viewed as an internal “surface” [31]. This surface error is seen to be smaller for LDA than for GGA [32] due to cancellation of errors within the exchange and correlation functions of the total energy [33]. Corrections for this error (referred to as the surface correction) for the different jumps in an

impurity containing system are not available in the literature. We use the computed migration barriers directly without any correction along with the vibrational pre-factors to obtain the jump frequencies and, subsequently, the impurity correlation factor.

Knowledge of the self-diffusion coefficient along with the calculated five jump frequencies and the impurity diffusion correlation factor allows one to obtain the impurity diffusion coefficient using Eq. (1). We use our first principles calculated Al self-diffusion coefficient [34] (without surface correction) to obtain the impurity diffusion coefficient in Al, illustrated in Figs. 3–6. In order to obtain the individual diffusion parameters, i.e. the diffusion pre-factor and the activation energy (Eq. (8)), the enthalpy and entropy of vacancy formation in pure Al and the enthalpy and entropy of vacancy–solute binding are calculated following standard thermodynamic relations [35] from the respective free energy expressions:

$$\Delta G_f^\circ = G_{PS} - \left( \frac{N-1}{N} \right) G_{IS}^{w_0}, \quad (9)$$

$$\Delta G_b = -(G_{IS}^{w_3} - G_{IS}^{w_4})$$

where  $G_{PS}$  is the free energy of the system without any vacancies and  $G_{IS}^{w_0}$ ,  $G_{IS}^{w_3}$  and  $G_{IS}^{w_4}$  are the free energies of

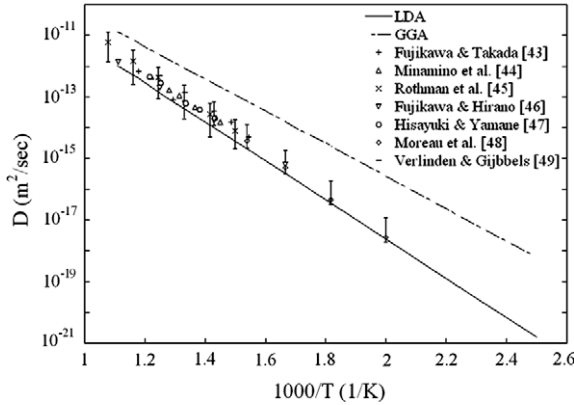


Fig. 3. Diffusion coefficient of Mg in fcc Al, comparing results from LDA and GGA with experimental data [43–49].

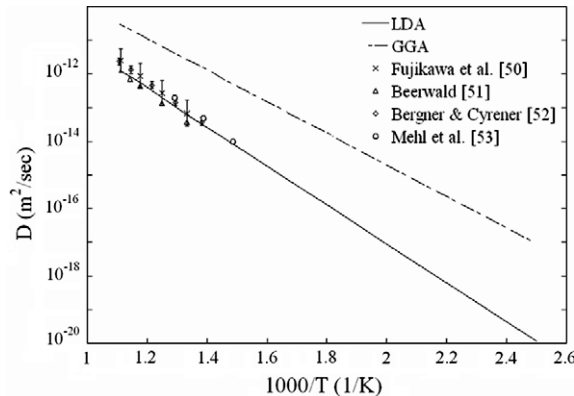


Fig. 4. Diffusion coefficient of silicon in fcc Al, comparing results from LDA and GGA with experimental data [50–53].

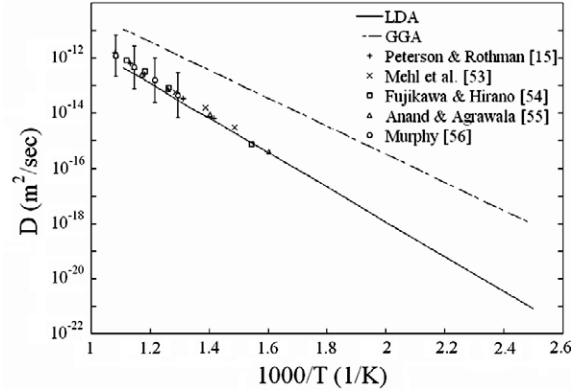


Fig. 5. Diffusion coefficient of copper in fcc Al, comparing results from LDA and GGA with experimental data [15,53–56].

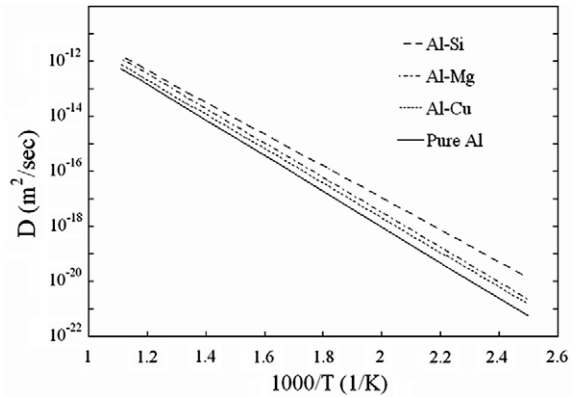


Fig. 6. Calculated diffusion coefficients of the impurities Mg, Si and Cu in Al in comparison with Al self-diffusion [9].

the initial states of the system for  $w_0$ ,  $w_3$  and  $w_4$  jumps, respectively. Again, no surface correction is added to the energetics of these states with vacancy. The free energies are calculated from the total energies ( $E_c$ ) and the vibrational frequencies, following the harmonic approximation [36]:

$$G(T) = E_c + k_B T \sum_q \sum_j \ln \left\{ 2 \sinh \left[ \frac{h\nu_j(q)}{2k_B T} \right] \right\}, \quad (10)$$

where the summation  $j$  is over the vibrational frequencies at each wave vector  $q$ .

#### 4. Results and discussion

Table 1 illustrates our calculated enthalpy and entropy of vacancy formation ( $\Delta H_f^\circ$ ,  $\Delta S_f^\circ$ ) and migration barrier ( $\Delta H_m^\circ$ ) in pure Al, as well as the enthalpy and entropy of solute–vacancy binding ( $\Delta H_b$ ,  $\Delta S_b$ ) and migration barrier ( $\Delta H_m$ ) of Mg, Si and Cu in Al. Where available, these quantities are compared with experimental measurements. We find a good agreement between our first principles calculated energetics and experimentally measured quantities. Our solute–vacancy binding energies (Table 1) also compare very well

Table 1

Calculated enthalpy and entropy of solute–vacancy binding ( $\Delta H_b$ ,  $\Delta S_b$ ) and solute migration barrier ( $\Delta H_m$ ) in system with impurity, and enthalpy and entropy of vacancy formation ( $\Delta H_f^\circ$ ,  $\Delta S_f^\circ$ ) and migration barrier ( $\Delta H_m^\circ$ ) in pure system, from LDA and GGA (without surface correction) in comparison with experimental data. Experimental migration barriers are deduced from activation energy and enthalpy of vacancy formation of the indicated references, assuming effect of correlation to be zero.

System	Enthalpy (eV) $\Delta H_f^\circ$ or $\Delta H_b$			Entropy ( $k_B$ ) $\Delta S_f^\circ$ or $\Delta S_b$			Migration barrier (eV) $\Delta H_m^\circ$ or $\Delta H_m$		
	LDA	GGA	Experimental	LDA	GGA	Experimental	LDA	GGA	Experimental
Al	0.71	0.55	$0.67 \pm 0.03$ [57]	1.21	1.18	1.10 [57]	0.58	0.52	$0.59 \pm 0.03$ [15]
Al–Mg	–0.07	–0.08	$-0.01 \pm 0.04$ [38]	–0.27	–0.32	$-0.10 \pm 0.50$ [38]	0.42	0.38	$0.67 \pm 0.07$ [45]
Al–Si	0.11	0.10	0.03 [38]	0.44	0.33	–2.00 [38]	0.55	0.479	$0.64 \pm 0.03$ [52]
Al–Cu	0.03	0.01	$\pm 0.12$ [38]	0.16	–0.51	$\pm 1.50$ [38]	0.57	0.476	$0.73 \pm 0.15$ [15]

with the previous first principles work of Wolverton [37], who used 64 atom supercells. We also report solute–vacancy binding entropies, however, there is a discrepancy with experimental data for these binding entropies. We suspect that this discrepancy may be due to the use of non-equilibrium quenching methods to obtain the experimental values of binding entropies, as discussed by Balluffi and Ho [38]. From Table 1 it can also be seen that the energetics of vacancy formation and atom migration in pure and impurity containing systems from LDA show better agreement with experimental data than those from GGA. Thus the results from the current work strongly support the conclusion from previous studies [32,33] that LDA has a smaller surface correction error than GGA.

The LDA calculated results for different jumps are given in Tables 2–4 for Mg, Si, and Cu impurities in Al and the impurity jump frequencies with respect to temperature are plotted in Fig. 2. It can be seen from these data that the lowest jump frequency ( $w_2$ ) of Cu in Al arises from the low  $v^*$  and high  $\Delta H_m$ . Similarly, the highest jump frequency of Mg in Al comes from the high  $v^*$  and low  $\Delta H_m$ . Further, from Eq. (2) we note that the greater the frequency of the impurity jump  $w_2$  and the lower the frequencies of jumps  $w_1$  and  $w_3$ , the lower the resulting impurity diffusion correlation factor, indicating a highly correlated motion of the impurity atom. This behavior is clearly reflected in the data tabulated in Table 4.

The various correlation factors are also plotted in Fig. 2 as a function of temperature. The Cu and Si diffusion cor-

Table 2

Calculated migration barriers  $\Delta H_m$  (eV) of the five jumps of different impurities from LDA.

Impurity	$w_0$	$w_1$	$w_2$	$w_3$	$w_4$
Mg	0.58	0.68	0.42	0.50	0.57
Si	0.58	0.52	0.55	0.66	0.55
Cu	0.58	0.38	0.57	0.61	0.59

Table 3

Calculated  $v^*$  (THz) for the five jumps of different impurities from LDA.

Impurity	$w_0$	$w_1$	$w_2$	$w_3$	$w_4$
Mg	16.6	21.8	18.6	13.3	17.1
Si	16.6	10.9	15.7	22.3	13.7
Cu	16.6	13.6	10.9	32.6	50.7

relation factors are nearly constant, while the Mg diffusion correlation factor increases with temperature. This temperature dependence effectively contributes a value of 0.068 eV to the activation energy of Mg diffusion in Al through the  $-k_B d \ln f_2/d(1/T)$  term in Eq. (8). The diffusion pre-factor ( $D_0$ ) and activation energy ( $Q$ ) are listed in Table 5, showing that there is a positive correlation between the size of the pre-factor and the size of the activation energy, specifically  $D_{0-\text{Mg}} > D_{0-\text{Cu}} > D_{0-\text{Si}}$  and  $Q_{\text{Mg}} > Q_{\text{Cu}} > Q_{\text{Si}}$ . This correlation is an example of the ubiquitous Meyer–Neldel compensation law [39] which is observed in many thermal activated Arrhenius-type processes. According to this law the diffusion coefficient of solutes with greater activation barriers is compensated by an increased diffusion pre-factor.

Our first principles calculated impurity diffusion coefficients are shown in Figs. 3–5. We find that the diffusion results from LDA are in excellent agreement with the experimental data. It should be noted that although the computed diffusion coefficients match well with experiments, there are discrepancies in the values of activation energy and diffusion pre-factor, as can be seen in Table 5. It has also been previously observed [11,40] that experimental activation barriers are consistently high compared with computational results. One possible reason for this discrepancy could be the experimental  $D_0$  and  $Q$  values being evaluated in small temperature ranges at high temperatures. However, further investigation of this discrepancy is warranted.

All of the calculated impurity diffusion coefficients for Si, Mg, and Cu in Al are plotted together in Fig. 6. From this figure we see a clear trend in diffusivities:  $D_{\text{Si}} > D_{\text{Mg}} > D_{\text{Cu}}$ . We wish to understand this trend. Prior studies [6,12–16] have described trends in solute diffusivity in terms of the solute atomic size and its excess valence or its solubility in the host element. However, these simple characteristics alone do not lend themselves to a simple explanation for our results: Si, the fastest diffuser of the three impurities, has an atomic radius ( $r$ ) intermediate between Mg and Cu [41]:

$$r_{\text{Mg}}(1.597\text{\AA}) > r_{\text{Si}}(1.392\text{\AA}) > r_{\text{Cu}}(1.284\text{\AA}).$$

Also, Cu, the slowest diffuser of the three, has a solubility ( $s$ ) in Al intermediate between Mg and Si:

$$S_{\text{Mg}} > S_{\text{Cu}} > S_{\text{Si}}$$

Table 4

Calculated five jump frequencies (Hz) and correlation factors for impurities diffusion along with jump frequency ratios entering Eq. (1). Results listed are for  $T = 400$  K using  $\Delta H_m$  from Table 2 and  $\nu^*$  from Table 3.

Impurity	$w_0$	$w_1$	$w_2$	$w_3$	$w_4$	$f_2$	$w_4/w_0$	$w_1/w_3$	$w_2/w_1$
Mg	8.31e5	5.81e4	9.18e7	6.60e6	1.09e6	0.16	1.3	0.01	1580
Si	8.31e5	3.35e6	1.86e6	1.24e5	1.76e6	0.66	2.1	27.02	0.56
Cu	8.31e5	2.39e8	7.17e5	6.72e5	1.87e6	0.99	2.2	354.98	0.003

Table 5

Diffusion pre-factor and activation energy from current LDA calculations to data in comparison with experiments and other theoretical calculations along with the assessed data by Du et al. [58]. For comparison,  $D_0$  is  $6.6e-6$  and  $Q$  is 1.29 for pure Al [9].

System property		Present	Experimental data	Other computations
$D_0$ (m <sup>2</sup> /sec)	Mg	1.19e-5	6.23e-6 [46]	1.24e-4 [1]
			1.24e-4 ± 0.22e-4 [45]	
			1e-4 [48]	
	Si	3.66e-6	2e-4 ± 0.66e-4 [43]	3.46e-5 [1]
			3.1e-5 [51]	
			3.5e-5 ± 5e-6 [52]	
	Cu	4.37e-6	6.47e-5 [15]	6.5e-5 [1]
			2.9e-5 [56]	
			4.44e-5 [58]	
$Q$ (eV)	Mg	1.27	1.19 [46]	1.35 [1]
			1.35 ± 0.05 [48]	1.2 [40]
			1.29 ± 0.015 [49]	1.13 [11]
	Si	1.15	1.41 ± 0.03 [43]	1.28 [1]
			1.33 [51]	1.0 [40]
			1.28 [52]	1.15 [11]
	Cu	1.25	1.4 ± 0.01 [15]	1.4 [1]
			1.35 ± 0.07 [56]	1.18 [11]
			1.39 [58]	

for temperatures just below the melting point of Al [42]. Trends in diffusivities have often been correlated with the trends in activation energies or even migration barriers [6,12,13,15]. However, we see that these simple descriptors do not even provide a qualitative explanation of the diffusion coefficients: Mg has a higher activation barrier than Cu and yet has a higher diffusivity than Cu; also, Mg has a lower migration barrier than Si but nevertheless has a lower diffusivity. Hence, for a quantitative understanding of the diffusion coefficients, or even a qualitative description of the trends, we find that it is crucial not only to assess the migration barriers and activation energies, but also to understand in detail the underlying factors entering the diffusion pre-factor, including the contribution of the correlation factor.

## 5. Conclusions

We have illustrated how impurity diffusion coefficients can be predicted directly from first principles without any empirical or fitting parameters. Specifically, we have used

DFT static (energy) and dynamic (vibrational) calculations to obtain the correlation factor through a five frequency model, the impurity jump frequency within the framework of transition state theory and the free energies of vacancy formation and vacancy–solute binding. We use our framework to calculate the impurity diffusion coefficients of Mg, Si and Cu in dilute fcc Al alloys. Using our approach we are able to obtain the values of individual diffusion parameters along with the diffusion pre-factor  $D_0$  and activation energy  $Q$ . The results match well with the experimental data. Both the diffusion pre-factor and the activation energy are key to a quantitative description of solute diffusivities, and we find that even the qualitative trends of diffusivity in Al may not always be inferred from activation energies or migration barriers alone.

## Acknowledgments

Work at Penn State was funded by the National Science Foundation (NSF) through Grants DMR-0510180 and DMR-0205232. C.W. acknowledges support from the US

Department of Energy under project DE-FG02-98ER 45721 and the US Automotive Materials Partnership (USAMP) through the US Council for Automotive Research (USCAR), Contract 07-1876. First principles calculations were carried out on the LION clusters at The Pennsylvania State University supported in part by NSF Grants DMR-9983532 and DMR-0122638 and in part by the Materials Simulation Center and the Graduate Education and Research Services at The Pennsylvania State University.

## References

- [1] Adams JB. *J Mater Res* 1989;4:102.
- [2] Blochl PE, Smargiassi E, Car R, Laks DB, Andreoni W, Pantelides ST. *Phys Rev Lett* 1993;70:2435.
- [3] Blochl PE, Van de Walle CG, Pantelides ST. *Phys Rev Lett* 1990;64:1401.
- [4] Frank W, Breier U, Elsasser C, Fahnle M. *Phys Rev Lett* 1996;77:518.
- [5] Janotti A, Krcmar M, Fu CL, Reed RC. *Phys Rev Lett* 2004;92:085901.
- [6] Krcmar M, Fu CL, Janotti A, Reed RC. *Acta Mater* 2005;53:2369.
- [7] Milman V, Payne MC, Heine V, Needs RJ, Lin JS, Lee MH. *Phys Rev Lett* 1993;70:2928.
- [8] Sandberg N, Magyari-Kope B, Mattsson TR. *Phys Rev Lett* 2002;89:065901.
- [9] Mantina M, Wang Y, Arroyave R, Wolverton C, Chen LQ, Liu ZK. *Phys Rev Lett* 2008;100:215901.
- [10] LeClaire AD, Lidiard AB. *Phil Mag* 1955;47:518.
- [11] Simonovic D, Sluiter MHF. *Phys Rev B* 2009;79:054304.
- [12] Janotti A, Krcmar M, Fu CL, Reed RC. *Phys Rev Lett* 2004;92:85901.
- [13] Lazarus D. *Phys Rev* 1954;93:973.
- [14] Swalin RA. *Acta Metall* 1957;5:443.
- [15] Peterson NL, Rothman S. *J. Phys Rev B* 1970;1:3264.
- [16] Neumann G, Hirschwald W. *Phys Status Solidi B* 1973;55:99.
- [17] Manning JR. *Diffusion kinetics for atoms in crystals*. Princeton (NJ): D. Van Nostrand; 1968.
- [18] Le Claire AD. *Correlation effects in diffusion in solids*. New York: Academic Press; 1970. p. 261.
- [19] Manning JR. *Phys Rev* 1964;136:A1758.
- [20] Wert C, Zener C. *Phys Rev* 1949;76:1169.
- [21] Vineyard GH. *J Phys Chem Solids* 1957;3:121.
- [22] Le Claire AD. *J Nucl Mater* 1978;69–70:70.
- [23] Kresse G, Furthmuller J. *Phys Rev B* 1996;54:11169.
- [24] Kresse G, Joubert D. *Phys Rev B* 1999;59:1758.
- [25] Blochl PE. *Phys Rev B* 1994;50:17953.
- [26] Ceperley DM, Alder BJ. *Phys Rev Lett* 1980;45:566.
- [27] Perdew JP, Burke K, Wang Y. *Phys Rev B* 1996;54:16533.
- [28] Henkelman G, Jonsson H. *J Chem Phys* 2000;113:9978.
- [29] Wei S, Chou MY. *Phys Rev Lett* 1992;69:2799.
- [30] Van de Walle A, Asta M, Ceder G. *CALPHAD* 2002;26:539.
- [31] Mattsson AE, Kohn W. *J Chem Phys* 2001;115:3441.
- [32] Vitos L, Ruban AV, Skriver HL, Kollar J. *Surf Sci* 1998;411:186.
- [33] Carling K, Wahnstrom G, Mattsson TR, Mattsson AE, Sandberg N, Grimvall G. *Phys Rev Lett* 2000;85:3862.
- [34] Mantina M. *Materials Science and Engineering*. Pennsylvania State University: University Park (PA); 2008. p. 232.
- [35] Hillert M. *Phase equilibria, phase diagrams and phase transformations: their thermodynamic basis*. Cambridge: Cambridge University Press; 1998.
- [36] Van de Walle A, Ceder G. *Rev Mod Phys* 2002;74:11.
- [37] Wolverton C. *Acta Mater* 2007;88:5867.
- [38] Balluffi RW, Ho PS. *Diffusion*. Metals Park (OH): American Society for Metals; 1973. p. 83.
- [39] Boisvert G, Lewis LJ, Yelon A. *Phys Rev Lett* 1995;75:469.
- [40] Sandberg N, Holmestad R. *Phys Rev B* 2006:73.
- [41] Wang Y, Curtarolo S, Jiang C, Arroyave R, Wang T, Ceder G, Chen LQ, Liu ZK. *CALPHAD* 2004;28:79.
- [42] Moffatt WG. *Handbook of binary phase diagrams*. Schenectady (NY): Genium Publishing; 1978.
- [43] Fujikawa S-I, Takada Y. *Defect Diffus Forum* 1997;143–147:409.
- [44] Minamino Y, Yamane T, Miyake T, Koizumi M, Miyamoto Y. *Mater Sci Technol* 1986;2:777.
- [45] Rothman SJ, Peterson NL, Nowicki LJ, Robinson LC. *Phys Status Solidi B* 1974;63:29.
- [46] Fujikawa S, Hirano K. *Mater Sci Eng* 1977;27:25.
- [47] Hisayuki K, Yamane T. *Z Metallkd* 1999;90:423.
- [48] Moreau G, Cornet JA, Calais D. *J Nucl Mater* 1971;38:197.
- [49] Verlinden J, Gijbels R. *Adv Mass Spectrom* 1980;8A:485.
- [50] Fujikawa S, Hirano K-I, Fukushima Y. *Metall Trans A* 1978;9:1811.
- [51] Beerwald AH. *Z Elektrochem Angew Phys Chem* 1939;45:789.
- [52] Bergner D, Cyrener E. *Neue Hutte* 1973;18:356.
- [53] Mehl RH, Rhines FN, Vonden Steinen KA. *Metals and Alloys* 1941;13:41.
- [54] Fujikawa S, Hirano K. *Defect Diff Forum* 1989;66–69:453.
- [55] Anand MS, Agrawala RP. *Phil Mag* 1972;26:297.
- [56] Murphy JB. *Acta Metall* 1961;9:563.
- [57] Erhart P, Jung P, Schultz H, Ullmaier H. *Landolt-Bornstein, New Series, Group III*. Berlin: Springer-Verlag; 1991.
- [58] Du Y, Chang YA, Huang B, Gong W, Jin Z, Xu H, Yuan Z, Liu Y, He Y, Xie F-Y. *Mater Sci Eng* 2003;2003:140.